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1972 J. Phys. A: Gen. Phys. 5 695

(http://iopscience.iop.org/0022-3689/5/5/011)

J. Phys. A: Gen. Phys., Vol. 5, May 1972. Printed in Great Britain

Spectral distribution of amplified spontaneous emission

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MS received 22 July 1971, in revised form 31 December 1971

Abstract. A theory is described which accounts for the dependence of the spectral distribution of amplified spontaneous emission on the length and inversion density of the active medium. For an inhomogeneously broadened line an unanticipated dip occurs in the plot of linewidth against length at lengths slightly greater than the critical length, but is explained by a simple physical analysis. Agreement occurs between the theory and that of Yariv and Leite in the regime over which their theory is expected to work. Observations by other workers on the spectral distribution of amplified spontaneous emission systems are discussed.

1. Introduction

In a recent series of papers (Peters and Allen 1971, 1972, Allen and Peters 1971a, 1971b, to be referred to as I, IV, II and III, respectively) the parametric behaviour of amplified spontaneous emission (ASE) has been theoretically predicted and experimentally verified. The theory is a rate equation approach interpreted in the light of a threshold condition and takes full account of the geometry of the ASE system.

In this paper the theory of papers I and III has been adapted and developed to include frequency dependence. The new theory allows the spectral distribution of ASE to be described. The only theoretical work on this seems to be that previously presented by Yariv and Leite (1963) who showed that under certain approximations the relative linewidth of the ASE, $\Delta\nu/\Delta\nu_D$, is given by

$$\frac{\Delta\nu}{\Delta\nu_D} = (\alpha L)^{-1/2}$$

for a column of atoms of length L and peak gain coefficient α. However, they did not take into account (i) critical length considerations, because the region concerned was well above threshold, (ii) the possibility of saturation, (iii) the geometry of the system which has the effect of varying the quantity of spontaneous emission that is usefully emitted at any point in the tube. Further it was necessary for them to assume that considerable line narrowing occurred to obtain their final result.

No experimental verification of Yariv and Leite's theory has been published for gaseous media, although they did demonstrate the validity of their theory with a semiconductor device. The spontaneous emission spectrum for a semiconductor, however, is homogeneously broadened while that of a gas is inhomogeneously broadened. So, some of the theory developed in this paper, particularly that involving critical length considerations, is not trivially applicable to the interpretation of their results.

Using the theory developed in this paper the spectral profiles of the 3·39 μm He–Ne radiation has been numerically computed for four tube lengths. Also the variation of relative linewidth as a function of tube length for the 3·39 μm He–Ne and 0·337 μm nitrogen systems has been deduced and plotted.

2. Theory

In the theory developed in papers I and III a very simplified form for the spontaneous emission spectrum and gain profile was taken, rather than the gaussian form typical of a gas system (viz rectangular profile). However, whereas the convolution of a spontaneous emission spectrum and gain profile of gaussian shape leads in general to a narrowing of the final spectral distribution of radiation, this does not occur if the frequency distribution is rectangular. So it is necessary to consider the spectral distribution of the spontaneous emission and gain profiles more carefully.

Before doing this, however, it is necessary to look closely at the cross section σ used previously in papers I and III. *If* σ is defined as the atomic resonance absorption cross section then the tacit assumption in writing the number of stimulated photons as $N\sigma n$, where N and n are total photon number and inversion density respectively, is that the spontaneous emission spectrum is homogeneously broadened. That is, a photon emitted anywhere on the emission profile can interact with *any* atom. In the case of a gas, motion of the atoms causes the emission line to be inhomogeneously broadened, with interaction only occurring if the atom's corresponding position on the gain profile is within a natural linewidth of the photon frequency. Thus what was actually defined in papers I and III was not the atomic resonance cross section σ', but an 'effective' cross section σ to allow for the fact that in a typical gas a photon cannot interact with just any atom. It can be shown that the following approximate relationship holds if the natural linewidth of the atom is very small compared to the Doppler width, $\Delta\nu_D$:

$$\sigma = \frac{\sigma'}{\Delta\nu_D}.$$

The theory developed in paper III can be extended to include a frequency term where all terms are defined exactly as before except they now apply to a particular frequency ν on the spontaneous emission and gain profile, $g(\nu)$. The equation for the rate of increase of the photon number $N(\nu, x)$ at frequency ν and position x in the tube has the following form:

$$\frac{\partial}{\partial x}N(\nu, x) = \{(n_2(\nu)-n_1(\nu))\sigma' - \beta a\}N(\nu, x) + \frac{An_2(\nu)a\Delta\Omega}{4\pi}$$

where the true atomic resonance absorption cross section has been used rather than the effective value and β is used to represent the loss term rather than ν, to avoid confusion with the frequency ν. By a similar procedure to that used in the earlier work we can show that

$$N(\nu, L) = \frac{Y(\nu, L)d^2}{4}\int_{L_c(\nu)}^{L}\frac{\exp(-X(\nu, L)y)\,\mathrm{d}y}{y^2}$$

where

$$X(\nu, L) = -\{(n_2(\nu)-n_1(\nu))\sigma' - \beta a\}$$

$$Y(\nu, L) = n_2(\nu)Aa.$$

Note the frequency dependence of the critical length at the lower limit of the integral. As the frequency ν moves away from the emission line centre, ν_0, the inversion that is available to be stimulated diminishes and consequently the appropriate critical length must increase.

The inversion and upper level population densities may also be written with their frequency dependence, and by an analogous procedure to that used in paper III it transpires that

$$n_2(\nu)-n_1(\nu) = \frac{Fg(\nu)}{1+E\Delta\nu_D\bar{N}_L(\nu)}$$

$$n_2(\nu) = \frac{Bg(\nu)+C\bar{N}_L(\nu)\Delta\nu_D}{1+E\Delta\nu_D\bar{N}_L(\nu)}$$

where B, C, E and F are the same constants as before. So the intensity of the ASE as a function of both L and ν may be expressed as

$$I(\nu, L) = \frac{I_{\text{spon}}(\nu, L)d^2}{4}\int_{L_c(\nu)}^{L} \exp\left\{-\left(\frac{K_1\bar{I}_L(\nu)\Delta\nu_D - K_2\Delta\nu_D g(\nu)}{1+K_3\bar{I}_L(\nu)\Delta\nu_D}\right)y\right\}y^{-2}\,dy \quad (1)$$

provided $L \geqslant L_c(\nu)$ and where K_1, K_2 and K_3 are defined exactly as in paper III.

It is shown in paper III that the spontaneous emission from the tube sides for the He–Ne system is effectively independent of L over the range of lengths employed. So $n_2(\nu)$ can be approximately expressed as $Bg(\nu)$ and the spontaneous intensity as

$$I_{\text{spon}}(\nu, L) \simeq I_{\text{spon}}g(\nu)$$

where I_{spon} is again that quantity defined in paper III.

To be able to predict the spectral distribution of the emitted radiation near threshold it is necessary to consider the unamplified spontaneous emission originating between $x = (L-L_c)$ and L in the tube. It can be shown that if the same detector parameters are kept as for the ASE measurements and light is collected from the same solid angle as that occupied by the ASE beam, then the spontaneous intensity at frequency ν may be expressed as

$$I_s(\nu, L) = \frac{I_{\text{spon}}d^2g(\nu)}{4}\left(\frac{2}{L_c(\nu_0)}-\frac{1}{L_c(\nu)}\right)$$

provided $L \geqslant L_c(\nu)$. If $L < L_c(\nu)$ then L must replace $L_c(\nu)$ in the expression.

The total intensity of the emitted radiation may now be expressed as a function of length and frequency as

$$I_T(\nu, L) = \frac{I_{\text{spon}}d^2g(\nu)}{4}\left[\int_{L_c(\nu)}^{L} \exp\left\{-\left(\frac{K_1\bar{I}_L(\nu)\Delta\nu_D - K_2\Delta\nu_D g(\nu)}{1+K_3\bar{I}_L(\nu)\Delta\nu_D}\right)y\right\}y^{-2}\,dy + \frac{2}{L_c(\nu_0)}-\frac{1}{L_c(\nu)}\right] \quad (2)$$

provided $L \geqslant L_c(\nu)$. The form of the gaussian distribution used is

$$g(\nu) = g(\nu_0)\exp\left\{-4\left(\frac{\nu-\nu_0}{\Delta\nu_D}\right)^2 \ln 2\right\}$$

where

$$g(\nu_0) = \frac{2(\ln 2)^{1/2}}{\sqrt{\pi}\Delta\nu_D}$$

when $g(\nu)$ is defined as the probability that a given transition will result in the emission of a photon with frequency ν and where

$$\int_0^{\infty} g(\nu)\,d\nu = 1.$$

Let the frequencies corresponding to the half intensity points of $I_T(\nu, L)$ be $\nu_0 \pm \nu_{1/2}$ so that the distribution width, $\Delta\nu$, is $2\nu_{1/2}$. By substituting the values ν_0 and $\nu_0 + \nu_{1/2}$ separately into the above equation, then by definition

$$I_T(\nu_0 + \nu_{1/2}, L) = \tfrac{1}{2} I_T(\nu_0, L).$$

It is true that this definition has no real meaning when the emission profile consists of a gaussian with a tiny component of ASE superimposed on its centre. However as the ASE grows although the resultant profile is complex such a definition becomes increasingly meaningful. Certainly when the ASE emission is an order of magnitude larger than the spontaneous emission there can be no doubt as to the physical meaning of such a definition. Using this definition $\Delta\nu$ may be expressed as

$$\Delta\nu = \Delta\nu_D \left\{ \frac{1}{\ln 2} \ln\left(2 \left[\int_{L_c(\nu_0+\nu_{1/2})}^{L} \exp\left\{ -\left(\frac{K_1 \bar{I}_L(\nu_0+\nu_{1/2})\Delta\nu_D - K_2 \Delta\nu_D g(\nu_0+\nu_{1/2})}{1 + K_3 \bar{I}_L(\nu_0+\nu_{1/2})\Delta\nu_D} \right) y \right\} \times y^{-2}\,dy + \frac{2}{L_c(\nu_0)} - \frac{1}{L_c(\nu_0+\nu_{1/2})} \right] \times \left[\int_{L_c(\nu_0)}^{L} \exp\left\{ -\left(\frac{K_1 \bar{I}_L(\nu_0)\Delta\nu_D - K_2 \Delta\nu_D g(\nu_0)}{1 + K_3 \bar{I}_L(\nu_0)\Delta\nu_D} \right) y \right\} y^{-2}\,dy + \frac{1}{L_c(\nu_0)} \right]^{-1} \right) \right\}^{1/2} \tag{3}$$

provided $L \geqslant L_c(\nu_0 + \nu_{1/2})$ and where

$$g(\nu_0 + \nu_{1/2}) = g(\nu_0) \exp\left\{ -\left(\frac{\Delta\nu}{\Delta\nu_D} \right)^2 \ln 2 \right\}.$$

If $L_c(\nu_0 + \nu_{1/2}) > L$ then the integral in the numerator must be put equal to zero (ie no ASE will occur at frequency $\nu_0 + \nu_{1/2}$) and L replaces $L_c(\nu_0 + \nu_{1/2})$ in the other term within the numerator.

It is necessary to express $L_c(\nu_0 + \nu_{1/2})$ as a function of $L_c(\nu_0)$, the quantity effectively measured in papers I and III. It is easy to show that

$$L_c(\nu_0 + \nu_{1/2}) = L_c(\nu_0) \frac{g(\nu_0)}{g(\nu_0 + \nu_{1/2})}.$$

Finally, $\bar{I}_L(\nu_0 + \nu_{1/2})$ and $\bar{I}_L(\nu_0)$ are deduced from $I(\nu_0 + \nu_{1/2}, L)$ and $I(\nu_0, L)$ which are not quantities that are easily accessible to measurement. It can be shown, however, that these may be expressed approximately in terms of the integrated ASE intensity $I(L)$ as follows:

$$I(\nu_0 + \nu_{1/2}, L) = \frac{I(L)}{2\Delta\nu} \qquad \text{and} \qquad I(\nu_0, L) = \frac{I(L)}{\Delta\nu}.$$

For a given system the appropriate constants can be determined by measuring the integrated ASE intensity $I(L)$ as a function of length L. This leaves only $\Delta\nu$ as an unknown quantity which can be deduced by an iterative procedure.

3. Theoretical predictions for the He–Ne and nitrogen systems

The fact that a certain length has to exist, for a given inversion density, before stimulated emission occurs implies that just above the threshold length $L_c(\nu_0)$, corresponding to maximum inversion, ASE occurs only in a very narrow spectral region. So the spectral profile of the radiation will follow closely that of the spontaneous emission spectrum except near ν_0 where a small 'bump' will occur. As the length of the tube is increased, the appropriate critical lengths for inversions farther from the line centre are achieved and the spectral width at the base of the bump increases as does its peak amplitude. As the peak amplitude of this bump becomes equal to the peak amplitude of the spontaneous emission spectrum, the width at halfheight of the total radiation is no longer determined by the spontaneous emission profile alone but increasingly by the ASE profile. Only when $L > 2L_c(\nu_0)$ will the spontaneous emission profile become completely modified over the whole Doppler width of the gain curve.

Evaluation of equation (3) presents a difficulty since the saturation parameter deduced from the work of paper III proves to be only an average $\bar{K}_3$. However to a good level of approximation it may be shown that $K_3\bar{I}_L(\nu)\Delta\nu_D$ may be replaced by $\bar{K}_3\bar{I}_L$. Figure 1 shows the spectral profiles of the 3·39 μm system of He–Ne for various tube lengths as predicted by numerical solution of equation (2). In figure 1, curve A shows the distribution when $L = L_c(\nu_0) = 180$ cm and only spontaneous emission

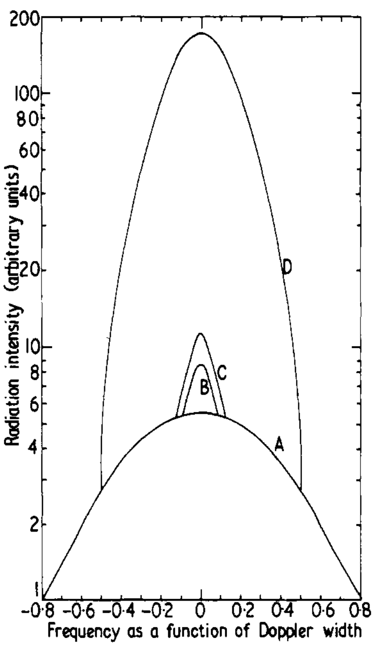

Figure 1. Theoretical spectral profile of the 3·39 μm transition for an inversion density of 15·8 units (ie $L_c(\nu_0) = 180$ cm) for tube lengths of 180 cm (A), 185 cm (B), 187 cm (C) and 360 cm (D) (logarithmic scale).

exists. In curve B, $L = L_c(\nu_0)+5$ cm and ASE occurs very near the line centre in a region less than 0·2 of the Doppler width, but the width at halfheight is still determined by the spontaneous emission profile. Curve C is similar, $L = L_c(\nu_0)+7$ cm, but now the width at halfheight is determined by the ASE rather than the spontaneous emission profile. So within 7 cm of $L_c(\nu_0)$ for the He–Ne system the predicted width at half-height has dropped to about 0·2 of the Doppler width, due to the effect of frequency dependent critical length. Finally, curve D shows that when $L = 2L_c(\nu_0)$, ASE occurs over the whole range $\Delta\nu_D$. When well above the threshold region critical length considerations become less important, because amplification is present over the major part of the emission profile, and the equation is not dissimilar in form to that of Yariv and Leite. In their experiment they varied the inversion and found that the emission line-width was modified, and that well above threshold the relationship between linewidth and the inversion was approximately as predicted. Because of the homogeneous broadening of the emission line in a semiconductor the critical length was not frequency dependent. Consequently the whole spectral width was narrowed; no 'bump' would be anticipated.

Figure 2 shows the predicted value of relative linewidth as a function of length for the 0·337 μm nitrogen system of Leonard (1965) as deduced from numerical computation of equation (3), using the values of K_1, K_2, $\bar{K}_3$ and $L_c(\nu_0)$ evaluated in paper III. Just above threshold the linewidth narrows sharply and then begins to increase again. After about $3L_c(\nu_0)$ the value approaches a maximum and then begins to fall again. The region where the definition of linewidth is rather dubious is shown by a dotted line. The full line is drawn at the point where the ASE component is an order of magnitude greater than the background. Certainly the dip in the linewidth against length plot is not simply the result of a bad definition of linewidth. At about $10L_c(\nu_0)$ the value approaches that given by the Yariv and Leite treatment to within a few per cent. For the remaining lengths up to 400 cm the two treatments give results which are at variance

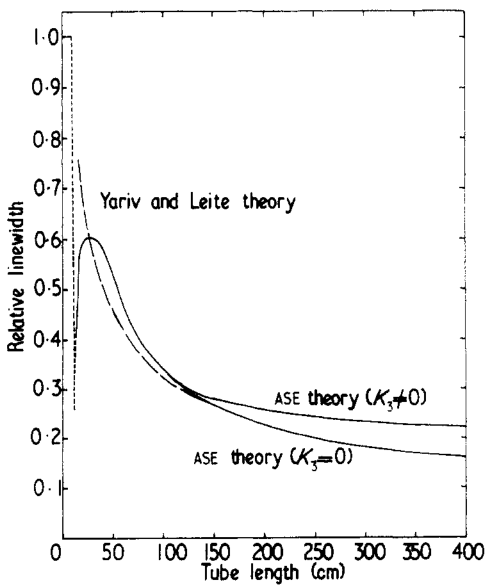

Figure 2. Theoretical curves for the variation of relative radiation linewidth with tube length according to the ASE and Yariv and Leite theories, for the 0·337μm nitrogen transition.

because the ASE theory has taken account of saturation. Perhaps a more meaningful comparison to make with the Yariv and Leite value is the ASE approach but with $K_3 = 0$, that is with the effects of saturation unaccounted for. For tube lengths < 100 cm the saturation term is of little importance and the ASE theory with K_3 finite and zero is essentially the same. For lengths > 100 cm the Yariv and Leite and ASE (with $K_2 = 0$) theories give results consistent to within 1 %. The assumptions made in interpreting Leonard's results were discussed in paper III and again imply a limitation to the degree of meaning which may be attached to them. Figure 3 shows the prediction of relative linewidth for the He–Ne system at 3·39 μm as the length is varied for an inversion of 15·8 units. As can be seen the linewidth seems to be approaching a maximum again at a length of about 3 $L_c(\nu_0)$.

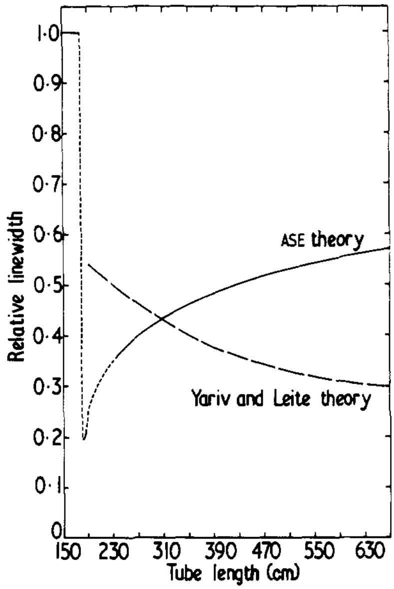

Figure 3. Theoretical curves for the variation of relative radiation linewidth with tube length according to the ASE and Yariv and Leite theories, for the 3·39 μm He–Ne transition.

One other effect that ought to be considered is power broadening, where the effect of the radiation on the atoms causes a broadening of the natural linewidth (see Bolwijn and Alkemade 1967). So as the length of inversion is increased, the emission spectrum narrows in the way described here because of stimulated emission. But at the same time the power of the increasing ASE is simultaneously causing the natural linewidth to increase. Thus conceivably there is a point where the latter is greater than the former, particularly in nitrogen where megawatt powers are obtainable, and then the observed linewidth of the radiation should essentially level out or even begin to slowly increase.

4. Discussion

Intuitively one might have expected that the spectral linewidth would always narrow with an increasing length of active medium but it has been shown that for an inhomogeneously broadened gain curve this is not the case. In a system where the effect of gain is very large it would be anticipated that the dip in the linewidth would be extremely narrow and could be missed. Certainly in all systems known to us we should anticipate that the dip would, in principle, be discernable.

It would be nice for completeness if, as with the other predictions in this series of papers, it had been possible to verify this theory experimentally. However, at the wavelength of 3·39 μm for He–Ne line it is exceedingly difficult to measure linewidth with sufficient accuracy; and even more so to align a tube to encompass the complete range up to $10L_c(\nu_0)$. So although the theory applied to He–Ne is unequivocal about its predictions, it does not seem to be a good system in which to attempt verification. The 0·614 μm neon transition is ideal from the point of view of optical analysis of linewidth, as Fabry–Perot interferometer fringes may easily be photographed or scanned with a photomultiplier. It is not, though, possible at this time to predict the dependence of linewidth on length using a longitudinal discharge tube because of pulse chasing effects, as discussed in paper III. In consequence verification, or otherwise, of this theory means the initiation of a whole new program of work. The only solution appears to be the construction of a cross-field device (see Leonard 1965) capable of excitation in incrementally increasing lengths, where pulse chasing effects are absent.

It should also be realized that while the theories derived in the present paper and in paper III may be expected to work well for CW systems (a steady state is assumed) it will only apply for pulsed systems when the population inversion is sensibly preserved along the length of tube for the entire duration of the ASE pulse. This occurs when the pulse duration τ is greater than the flight time, L/c, of a photon along the tube. When $\tau < L/c$, the population inversion will decay before the ASE pulse has traversed the medium and absorption of the ASE pulse will occur (see Leonard's work on neon 1967). Thus, Leonard's N_2 system with its pulse duration of about 20 ns and flight time of about 10 ns works well, whilst in his neon system the pulse duration was just less than the flight time.

When this theory is applied to a pulse system its validity is limited in another way. A pulse of duration τ, Fourier transforms to give a minimum frequency spread of $\Delta\nu_F \sim 1/\tau$. So in the systems of Shipman (1967) and Ericsson and Lidholt (1968), where pulse durations of 4 ns and 0·7 ns, respectively, were obtained, minimum relative linewidths of 0·1 and 0·6 would occur. This limit cannot be discussed within the framework of this theory since the duration of the pulse is not taken into account. Effectively $\tau \to \infty$ in this theory. So the linewidth predicted by the ASE theory must always be in the region $\Delta\nu \gg \Delta\nu_F$ to be valid. In the case of Leonard's nitrogen system $\Delta\nu/\Delta\nu_D \sim 0{\cdot}2$ while $\Delta\nu_F/\Delta\nu_D \sim 0{\cdot}02$. Taking the form of Yariv and Leite's theory, which is a rough approximation to the ASE theory for the maximum lengths investigated, the frequency condition for the ASE theory to be valid becomes

$$\frac{\Delta\nu_D}{(\alpha L)^{1/2}} \gg \frac{1}{\tau}.$$

In a recent paper by Korolev *et al* (1970) it has been suggested that ASE in neon exhibits mode structure, with a separation typical of that observed in a laser. The modes of a laser are a property of the cavity, rather than the active medium, and it is difficult to see why modes of comparable separation should exist in an ASE system. Certainly mode structure of ASE is a meaningful concept as originally used by us (Allen and Peters 1970) in deriving the onset criterion for ASE to occur. However, these modes are essentially free space modes, with a separation of about 10 Hz for the wavelength and tube dimensions appropriate to the neon system. The mode structure was apparently at first assumed to be isotopic but the results were shown to be inconsistent with such a possibility. It was not positively demonstrated that the ASE itself was the cause, nor was any explanation given as to how mode structure could arise. Indeed the authors' report that they found no conclusive dependence of the splitting on tube current, gas pressure or tube geometry; a most unusual result if the mode structure is associated with the ASE.

There is another more likely origin for the ring splitting observed and that is mode structure associated with the analysing Fabry–Perot interferometer. At first sight the structure does not appear to be due to axial modes as the separation was about 1000 MHz for their system. Goldsborough (1964) has analysed transverse mode structure and his work implies that the separation of adjacent modes for Korolev's system would be between 0 and 500 MHz depending upon the radius of curvature of the mirrors. Korolev *et al* (1970) do not quote a value for the mirror curvature but the values they observed for mode separation, 150–400 MHz, are certainly consistent with the excitation of transverse modes in the analyser.

The problem arises in both interpretations as to how radiation of 100–200 MHz bandwidth, the value quoted by Korolev *et al*, can excite several modes with a separation of 150–400 MHz. The answer here appears to be that Korolev *et al* deduced the ASE linewidth from the pulse duration but as discussed earlier this only applies a lower limit to the spread of frequencies; the actual linewidth may be much larger and determined by the gain of the system.

As has been discussed power broadening effects would cause an increase in the ASE linewidth, and also just above threshold the linewidth begins to grow again with increasing length (or inversion). So if the experiment were performed in either of these two regions then the increase in the number of modes which Korolev *et al* observed, could be accounted for.

The use of ASE as a wavelength standard was suggested by Rigden and White (1963) with its first observation at 3·39 μm in He–Ne. Unlike the resonant modes of a laser cavity, which shift in absolute frequency with the slightest change in cavity length, the ASE profile is always centred on the atomic resonance which is relatively insensitive to excitation conditions. Egorov and Plekhotkin (1969) observed a small wavelength shift in an ASE system compared to that of emission from a glow discharge where conditions are such that the transition is practically unperturbed. The shift was due to the perturbation induced by the electron current, and its maximum magnitude was approximately 300 MHz (ie $\sim 0{\cdot}1\ \Delta\nu_D$). This obviously will put a limit on the accuracy of an ASE beam used as a wavelength standard if the excitation conditions are not to be specified. The presence of the strong linewidth dip described in this paper shows that a short length of amplifying media or a low inversion density can give a very narrow spectral profile even though the output intensity will not be very great. The latter may prove useful in reducing shifts due to the perturbing effects of large electric fields.

Acknowledgments

One of us (GIP) wishes to thank the Science Research Council for providing a Research Studentship.

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